CEE 361/ MAE 325 / MSE 331 /CEE 513 : Introduction to finite element methods

Lectures: Tue Thu 11:00AM-12:20PM, Friend Center 008

Precept: Mon 7:30PM-8:30PM, Friend Center 110

Overview

The course introduces fundamental concepts and technologies of primal finite element methods for linear elliptic boundary value problems.

The course covers an overview of finite element methods for a one-dimensional model problem including the weak, Galerkin and matrix forms, error analysis and superconvergence. The direct stiffness method of structural analysis is introduced to present the notion of assembly.

Extension of finite element methods to multiple dimensions are carried out, first for second order scalar valued equations, such as the heat equation and Darcy's flow in porous materials, and later extended to vector valued equations such as the elasticity equations. The course then concludes with the C^0 approach to plates and beams and contrasts it with matrix structural analysis approaches.

The element formulations and data structures, isoparametric interpolations, locking issues, analysis of errors and convergence of approximations, as well as treatment of constraints and variational crimes will all be discussed.

Pre-requisites

The students should have a background in multivariable calculus (MAT 202) and linear algebra (MAT 201). Prior programming experience (in particular with `python`) will be beneficial but not necessary.

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Assistant: Vivek Kumar
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Office hours: Mon 8:30pm - 10:20pm, Friend Center 110

Course website: `finiteelements.princeton.edu`

The course website will contain homework, notes, and other relevant course information. To access documents you will be prompted for a login. Use the following:
username: `fea`
password: `galerkin`.

Textbooks: Required textbook:

T. J. R. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. 1987.

Suggested readings:

J. H. Prévost and S. Bagrianski. *An Introduction to Matrix Structural Analysis and Finite Element Methods*. WORLD SCIENTIFIC, Mar. 2017, pp. i–xiii. ISBN: 978-981-320-677-9.

Grading: Letter grades A-F. Grades will be based on homework assignments (40%), a midterm & final exam ($2 \times 15\% = 30\%$) and a final project (30%).

The final grade for the class is computed with both an absolute and relative (curve) scale. If you get above or equal to:

- 98% of the maximum score you get an A+
- 90% of the maximum score you get an A
- 85% of the maximum score you get an A-
- 80% of the maximum score you get an B+
- 75% of the maximum score you get an B
- 70% of the maximum score you get an B-
- 65% of the maximum score you get an C+
- 60% of the maximum score you get an C
- 55% of the maximum score you get an C-
- 50% of the maximum score you get an D+
- 45% of the maximum score you get an D

If you get less than 45% of the maximum score you get an F. Simultaneously, at least 25% of the class will get an A, at least 50% of the class will get a B or higher, and at least 75% of the class will get a C or higher, as long as the minimum of 45% of the maximum total score is achieved. This distribution will be achieved by uniformly shifting everyone's grade up by the same amount.

Homework: Homework assignments will consist of a mixture of theory as well as computational exercises. Homework will generally be assigned on Thursday and due a week later. There will be 6-10 homework assignments. Students enrolled in CEE 513 will need to typeset homework in LaTeX. Students not enrolled in CEE 513 that typeset homework assignments in LaTeX will receive 5% bonus points. Irrespective, homework should be neatly and clearly presented. If the quality of presentation is not satisfactory, the homework will be receive a 0%. We will not accept late homework, but the lowest grade on homework assignment will be dropped. Each homework will be equally weighted.

Exams: The exams will be closed-book and closed-notes. They will focus on the theory. The midterm exam will be on October 26th and the final exam date is TBD.

Project: The final project will allow the student to choose a topic of interest to perform both theoretical and computational work using the tools acquired in the course. A proposal for the project will be due the week after Fall Recess. A mid-term project report will due 4 weeks after the proposal is returned. A final project report will be due on Dean's Date. More details on the project will be distributed in a separate document.

Grad. Expect.: Students taking the CEE 513 will have additional homework problems, (homework and project) and exams.

Attendance: It is your responsibility to attend lectures and precepts and take notes. Not all material discussed in lecture is available in the suggested readings.

Piazza: This term we will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from classmates, the AI, and myself. Rather than emailing questions to the teaching staff, I encourage you to post your questions on Piazza.

The piazza link for the course is:
`http://piazza.com/princeton/fall2017/cee361mae325mse331cee513f17`.

Course Outline

1. Math Preliminaries
   (a) Tensor algebra and tensor calculus
   (b) Review of partial differential equations
2. Direct Stiffness Methods
   (a) Truss Equation
   (b) Beam Equation
   (c) Global Assembly
   (d) Boundary conditions
3. One-dimensional Finite elements
   (a) Intro to the Calculus of variations
   (b) Strong and weak form
   (c) Galerkin approximation
   (d) Matrix form
   (e) Shape functions
   (f) Numerical integration
   (g) Error analysis
4. Two and three space dimensions
   (a) Review of tensors calculus
   (b) Extension of the notions of strong, weak, and matrix forms
   (c) Hexahedral elements
   (d) Simplicial elements
   (e) Isoparametric interpolations
5. C^0 approach to beams and shells
   If Time Permits:
6. Time dependent problems
   (a) Generalized θ method
   (b) Stability of the θ-method

7. Analysis of Finite Element Methods
    (a) Best approximation and error estimates
    (b) Consistency & stability
    (c) Locking issues

Additional topics - time permitting:

1. Constraints
    (a) Lagrange multipliers
    (b) Penalty methods
2. Finite elements contrasted to direct stiffness methods (2)
    (a) Truss elements
    (b) Beams

Tentative Schedule:

Monday	Tuesday	Thursday
Sep 11th **1**	12th **2**	14th **3** Course Overview
18th **4** Precept	19th **5** Review of PDEs & Tensors	21st **6** Review of PDEs & Tensors Homework: # 1 Out
25th **7** Precept	26th **8** Direct Stiffness Method	28th **9** Direct Stiffness Method Homework: # 1 Due - #2 Out
Oct 2nd **10** Precept	3rd **11** Direct Stiffness Method	5th **12** Direct Stiffness Method Homework: # 2 Due - #3 Out

Monday	Tuesday	Thursday
9th **13** Precept	10th **14** 1-D Finite Elements: Strong and weak form	12th **15** 1-D Finite Elements: Galerkin Approximation Homework: # 3 Due - # 4 Out
16th **16** Precept	17th **17** 1-D Finite Elements: Element View	19th **18** 1-D Finite Elements: Shape Functions & numerical integration Homework: # 4 Due - # 5 Out
23rd **19** Precept	24th **20** 1-D Finite Elements: Error analysis	26th **21** **Midterm Exam**
30th Fall Recess	31st Fall Recess	Nov 2nd Fall Recess
6th **22** Precept	7th **23** n-D Finite Elements: Review of tensors & tensor calculus **Project Proposal Due**	9th **24** n-D Finite Elements: Strong, weak, and matrix forms Homework: # 5 Due - # 6 Out
13th **25** Precept	14th **26** n-D Finite Elements: Hexahedral elements	16th **27** n-D Finite Elements: Hexahedral elements Homework: # 6 Due - # 7 Out
20th **28** Precept	21st **29** n-D Finite Elements: Simplicial elements	23rd Thanksgiving Break

Monday	Tuesday	Thursday
27th **30** Precept	28th **31** n-D Finite Elements: Simplicial elements	30th **32** C^0 approach to beams and shells Homework: # 7 Due - # 8 Out
Dec 4th **33** Precept	5th **34** C^0 approach to beams and shells	7th **35** Time dependent problems: Generalized θ method Homework: # 8 Due - # 9 Out
11th **36** Precept	12th **37** Time dependent problems: Stability of the θ-method **Mid-term project report due**	14th **38** Analysis of Finite Element Methods Homework: # 9 Due - # 10 Out
18th Winter Recess	19th Winter Recess	21st Winter Recess
25th Winter Recess	26th Winter Recess	28th Winter Recess
Jan 1st Winter Recess	2nd Winter Recess	4th Winter Recess
8th Reading Period	9th Reading Period	11th Reading Period
15th **Project report due** Reading Period	16th Reading Period Dean's Date	18th Reading Period

Monday	Tuesday	Thursday
22nd Final Examination	23rd Final Examination	25th Final Examination