CEE 361/ MAE 325 / MSE 331 :
Matrix Structural Analysis and Introduction to Finite-Element Methods

Lectures: Tue Thu 11:00AM-12:20PM, Friend Center 008

Precept: Wed 7:30PM-8:30PM, Friend Center 008

Overview

The course introduces fundamental concepts and technologies of primal finite element methods for linear elliptic boundary value problems.

The course covers an overview of finite element methods for a one-dimensional model problem including the weak, Galerkin and matrix forms, error analysis and superconvergence. The direct stiffness method of structural analysis is introduced to present the notion of assembly.

Extension of finite element methods to multiple dimensions are carried out, first for second order scalar valued equations, such as the heat equation and Darcy's flow in porous materials, and later extended to vector valued equations such as the elasticity equations. The course then concludes with the C^0 approach to plates and beams and contrasts it with matrix structural analysis approaches.

The element formulations and data structures, isoparametric interpolations, locking issues, analysis of errors and convergence of approximations, as well as treatment of constraints and variational crimes will all be discussed.

Pre-requisites

The students should have a background in multivariable calculus (MAT 202) and linear algebra (MAT 201). Prior programming experience (in particular with `python`) will be beneficial but not necessary.

Instructor:
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Office hours: Wed 1:30pm - 3:30pm, EQuad E324

Assistant:
Isabel Morris
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Office hours: Wed 8:30pm - 10:20pm, Friend Center 008

Course website: `finiteelements.princeton.edu`

The course website will contain homework, notes, and other relevant course information. To access documents you will be prompted for a login. Use the following:
username: `bubnov`
password: `galerkin`.

Textbooks: Suggested readings:

T. J. R. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. 1987. DOI: 620/âĂŇ.001/âĂŇ51535. URL: http://books.google.com/books?id=yarmSc7ULRsC.

J. H. Prévost and S. Bagrianski. *An Introduction to Matrix Structural Analysis and Finite Element Methods*. WORLD SCIENTIFIC, Mar. 2017, pp. i–xiii. ISBN: 978-981-320-677-9. DOI: 10.1142/10358. URL: http://www.worldscientific.com/doi/abs/10.1142/9789813206793%7B%5C_%7Dfmatter%20http://www.worldscientific.com/worldscibooks/10.1142/10358.

Grading: Letter grades A-F. Grades will be based on homework assignments (40%), a midterm & final exam ($2 \times 15\% = 30\%$) and a final project (30%).

The final grade for the class is computed with both an absolute and relative (curve) scale. If you get above or equal to:

- 98% of the maximum score you get an A+
- 90% of the maximum score you get an A
- 85% of the maximum score you get an A-
- 80% of the maximum score you get an B+
- 75% of the maximum score you get an B
- 70% of the maximum score you get an B-
- 65% of the maximum score you get an C+
- 60% of the maximum score you get an C
- 55% of the maximum score you get an C-
- 50% of the maximum score you get an D+
- 45% of the maximum score you get an D

If you get less than 45% of the maximum score you get an F. Simultaneously, at least 25% of the class will get an A, at least 50% of the class will get a B or higher, and at least 75% of the class will get a C or higher, as long as the minimum of 45% of the maximum total score is achieved. This distribution will be achieved by uniformly shifting everyone's grade up by the same amount.

Homework: Homework assignments will consist of a mixture of theory as well as computational exercises. Homework will generally be posted on Monday and due a week later (the Monday after by 11:59pm). Homework will be submitted electronically through https://blackboard.princeton.edu dropbox. There will be 10 homework assignments. Students that typeset homework assignments in LaTeX will receive 5% bonus points. Irrespective, homework should be neatly and clearly presented. If the quality of presentation is not satisfactory, the homework will receive a 0%. We will not accept late homework, but the lowest grade on homework assignment will be dropped. Each homework will be equally weighted.

Exams: The mid-term exam will be closed-book and closed-notes. It will focus on the theory and will be on October 25th.

Project: The final project will be an open ended assignment where, as a future Engineer, you will be assigned a challenge and are expected to find the best possible solution leveraging the skills you acquired in the course. More details will follow.

Precepts: These are interactive problem sessions where you get a chance to practice your skills while interacting with other students and the AI.

Attendance: It is your responsibility to attend lectures and precepts while taking notes. Not all material discussed in lecture is available in the suggested readings.

Piazza: This term we will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from classmates, the AI, and myself. Rather than emailing questions to the teaching staff, I encourage you to post your questions on Piazza.

The piazza link for the course is:
`http://piazza.com/princeton/fall2018/cee361mae325mse331cee513f18`.

Course Outline

1. Math Preliminaries
   (a) Tensor algebra and tensor calculus
   (b) Review of partial differential equations
2. Direct Stiffness Methods
   (a) Truss Equation
   (b) Beam Equation
   (c) Global Assembly
   (d) Boundary conditions
3. One-dimensional Finite elements
   (a) Intro to the Calculus of variations
   (b) Strong and weak form
   (c) Galerkin approximation
   (d) Matrix form
   (e) Shape functions
   (f) Numerical integration
   (g) Error analysis
4. Two and three space dimensions
   (a) Review of tensors calculus
   (b) Extension of the notions of strong, weak, and matrix forms
   (c) Hexahedral elements
   (d) Simplicial elements
   (e) Isoparametric interpolations
5. C^0 approach to beams and shells
   If Time Permits:
6. Time dependent problems
   (a) Generalized θ method
   (b) Stability of the θ-method
7. Analysis of Finite Element Methods
   (a) Best approximation and error estimates
   (b) Consistency & stability
   (c) Locking issues

Additional topics - time permitting:

1. Constraints
    (a) Lagrange multipliers
    (b) Penalty methods
2. Finite elements contrasted to direct stiffness methods (2)
    (a) Truss elements
    (b) Beams

Tentative Schedule:

Monday	Tuesday	Wednesday	Thursday
Sep 10th **1**	11th **2**	12th **3**	13th **4** Course Overview
17th **5**	18th **6** Review of PDEs & Tensors	19th **7** Precept	20th **8** Review of PDEs & Tensors
24th **9** Homework: # 1 Out	25th **10** Direct Stiffness Method Trusses in 1D	26th **11** Precept	27th **12** Direct Stiffness Method Trusses in 1D
Oct 1st **13** Homework: # 1 Due - 2 Out	2nd **14** Direct Stiffness Method Trusses in 2D	3rd **15** Precept	4th **16** Direct Stiffness Method Trusses in 2D
8th **17** Homework: # 2 Due - 3 Out	9th **18** Direct Stiffness Method Beams in 1D	10th **19** Precept	11th **20** Direct Stiffness Method Beams in 2D
15th **21** Homework: # 3 Due - 4 Out	16th **22** 1-D Finite Elements: Strong and weak form	17th **23** Precept	18th **24** 1-D Finite Elements: Galerkin Approximation
22nd **25** Homework: # 4 Due - 5 Out	23rd **26** 1-D Finite Elements: Element View	24th **27** Precept	25th **28** **Midterm Exam** Final project posted
29th Fall Recess	30th Fall Recess	31st Fall Recess	Nov 1st Fall Recess

Monday	Tuesday	Wednesday	Thursday
5th **29** Homework: # 5 Due - 6 Out	6th **30** 1-D Finite Elements: Shape Functions & numerical integration	7th **31** Precept	8th **32** 1-D Finite Elements: Error analysis
12th **33** Homework: # 6 Due - 7 Out	13th **34** n-D Finite Elements: Review of tensors & tensor calculus	14th **35** Precept	15th **36** n-D Finite Elements: Strong, weak, and matrix forms
19th **37** Homework: # 7 Due - 8 Out	20th **38** n-D Finite Elements: Hexahedral elements	21st Thanksgiving Break	22nd Thanksgiving Break
26th **39** Homework: # 8 Due - 9 Out	27th **40** n-D Finite Elements: Hexahedral elements	28th **41** Precept	29th **42** n-D Finite Elements: Simplicial elements
Dec 3rd **43** Homework: # 9 Due - 10 Out	4th **44** n-D Finite Elements: Simplicial elements	5th **45** Precept	6th **46** C^0 approach to beams and shells
10th **47** Homework: # 10 Due	11th **48** C^0 approach to beams and shells	12th **49** Precept	13th **50** Time dimension Final Project Due