HOMEWORK 3

CEE 361-513: Introduction to Finite Element Methods

Due: October 19, 2018

NB: Students taking CEE 513 must complete all problems. All other students will not be graded for problems marked with $\star$, but are encourage to attempt them anyhow.

PROBLEM 1

Consider the truss shown below. Foreach node $z = 1, 2, 3$ we have associated coordinates $\boldsymbol{q}_z$ and associated global degrees of freedom $\boldsymbol{u}_z$, where both $\boldsymbol{q}$ and $\boldsymbol{u}$ are vectors.

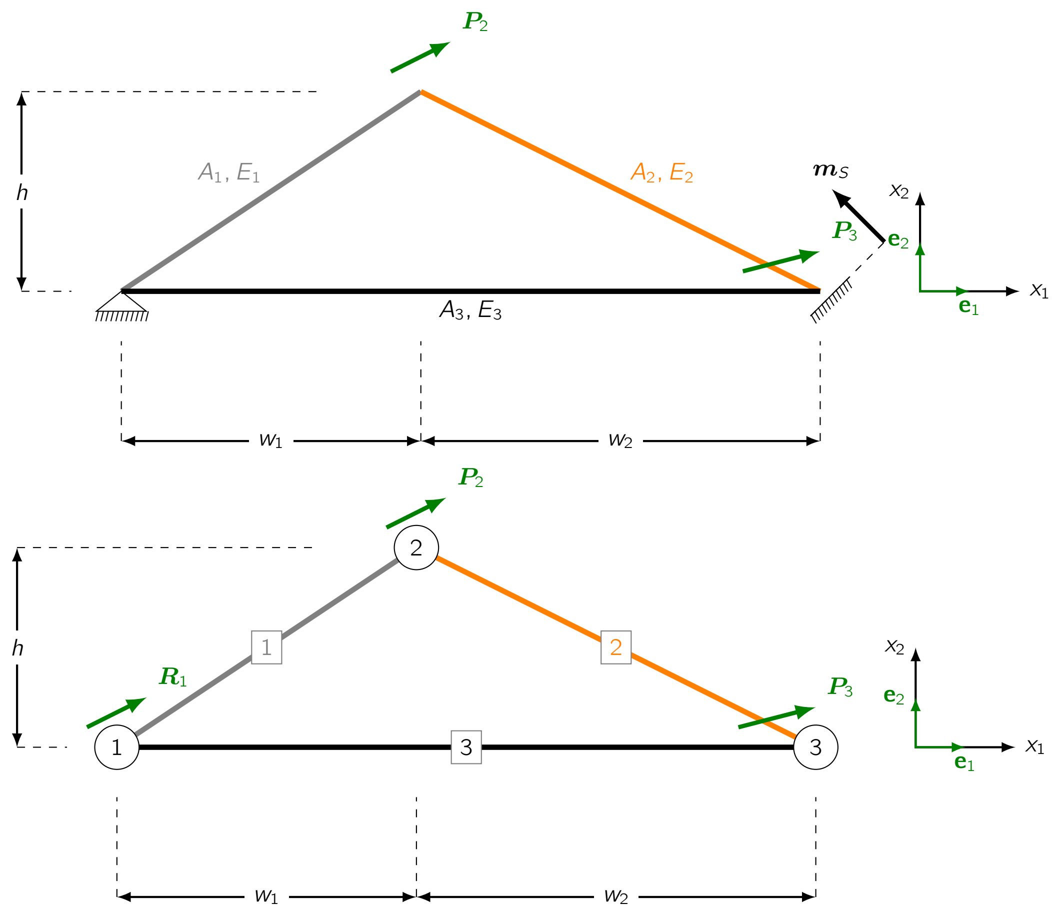

Figure 1: The system of uniaxial rods

1. For each element write the internal forces as the matrix vector operation of the *local element stiffness* and the *local degrees of freedom*.
2. For each element write the internal forces as the matrix vector operation of the *local element stiffness* and the *GLOBAL degrees of freedom* using the connectivity array.
3. For each node write the equilibrium equations in terms of the external forces $\boldsymbol{P}_k$, $k = 2, 3$, the reactions $\boldsymbol{R}_1$, and the internal forces $\boldsymbol{f}^e_{i,j}$.
4. Let $k_i = A_i E_i / \ell_i$ for $i = 1 \ldots 3$. Write down the equilibrium equations in matrix form. Namely, as we did in class, write the equilibrium equations with a load vector containing reactions and external forces,

denoted it by {P}, the stiffness matrix denoted by $[K]$, and the vector of displacements $\{U\}$ such that

$$[K]\{U\} = \{P\}.$$

5. At node 1 we prevent the truss from moving. At node 3 we allow the truss to move along a plane whose unit normal is $\boldsymbol{m}_2$. Apply the aforementioned conditions to $[K], \{P\}$.

PROBLEM 2

Consider the truss of Problem 1. Let $w_1 = 7, w_2 = 10, h = 7$, and $A_1E_1 = 10, A_2E_2 = 20, A_3E_3 = 30$. Further let $\boldsymbol{P}_2 = 10\mathbf{e}_1 + 5\mathbf{e}_2, \boldsymbol{P}_3 = 2\mathbf{e}_1 + 5\mathbf{e}_2$.

1. Construct the connectivity array for the truss drawn.
2. Write in `python` a function `local_to_global_dof` that takes as arguments (1) the connectivity array, (2) the element number, and (3) the local degree of freedom (i or j) and returns the corresponding global degree of freedom. Namely:
   `local_to_global_dof( connectivity_array, element_number, local_dof )`
3. Write in `python` a function `element_stiffness` that takes as arguments (1) the element Young's modulus, (2) the element cross sectional area, (3) the $\boldsymbol{q}_i$ coordinate, (4) the $\boldsymbol{q}_j$ coordinate, and returns the element stiffness. Namely:

   `element_stiffness( youngs_modulus, area , q_i, q_j )`
4. Similarly to the last homework, write a loop that assembles the global element stiffness matrix.
5. As before, at node 1 we prevent the truss from moving hence $\boldsymbol{u}_1 = 0$. At node 3 we allow the truss to move along a plane whose unit normal is $\boldsymbol{m}_2 = -\sin(\pi/4)\mathbf{e}_1 + \cos(\pi/4)\mathbf{e}_2$. Apply the aforementioned conditions to $[K], \{P\}$. (hint: you will have to introduce an additional unknown λ).
6. What are the displacements of the nodes ?
7. What are the reactions $\boldsymbol{R}_1, \lambda$?
8. Plot in `python` the deformed shape of the truss.

PROBLEM 3 $\star$

Repeat the steps of Problem 2 for the truss shown below. At nodes 1, 3, 4, 6 the truss is not allowed to move. At node 5 we have a load $\boldsymbol{P}_5 = -5\mathbf{e}_2$ and at node 2 we have load $\boldsymbol{P}_2 = 5\mathbf{e}_2$. Further let $AE = 10$. Plot in `python` the deformed configuration of the truss.